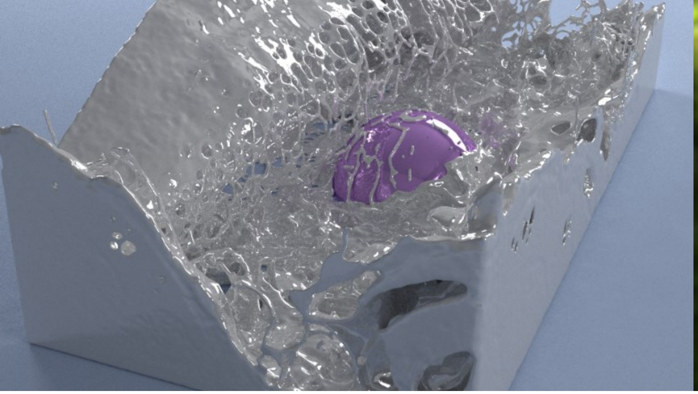

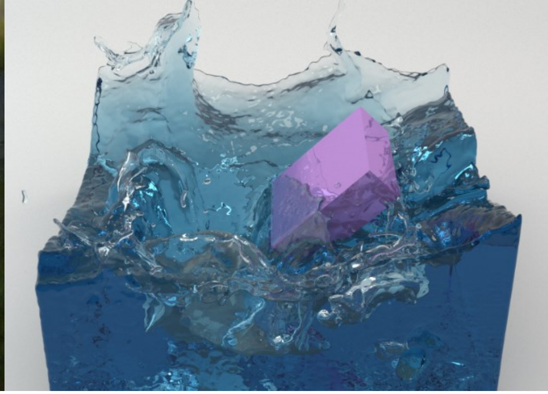

Advanced Hybrid Particle-Grid Method with Sub-Grid Particle Correction

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Introduction

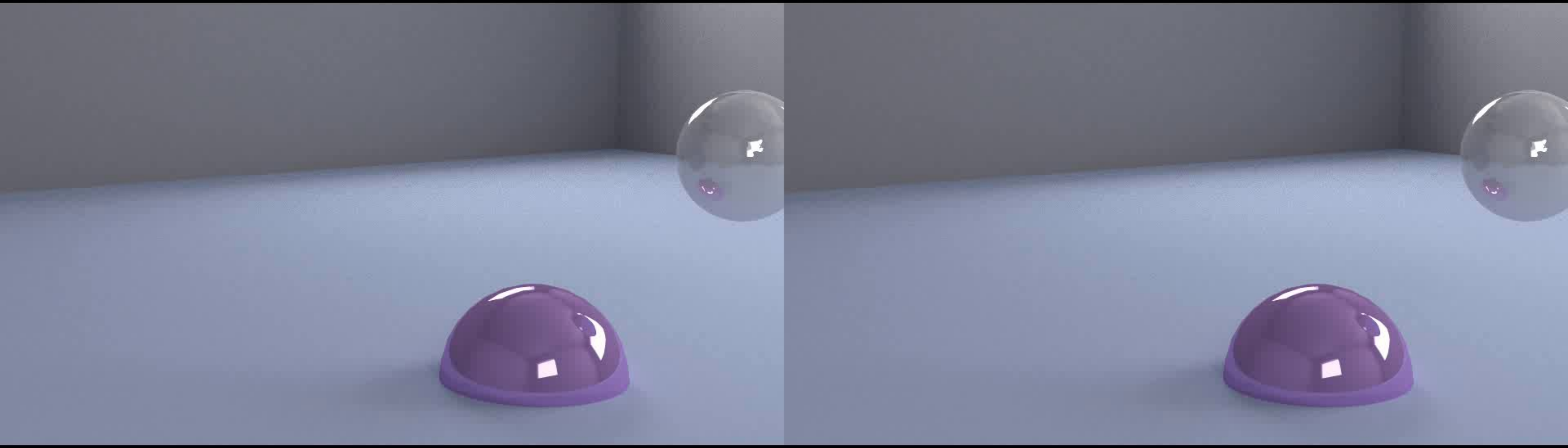
- Particle-in-cell (PIC) / fluid implicit particle (FLIP)
  - Hybrid particle-grid approach
  - Benefits from both sides, particle and grid
  - Easy implementation
  - Popular method in computer graphics

- Just good?
  - Disturbance in distribution of particles
  - Bumpy surface
  - Volume loss

- FLIP + grid-based particle correction: FLIP-GPC

Introduction (cont'd)

smooth surface and dynamic liquid motion



FLIP

FLIP-GPC
(our method)

Related Work

- Zhu and Bridson, 2005, SIGGRAPH,
  "Animating sand as a fluid"

- Ando and Tsuruno, 2011, SCA,
  "A particle-based method for preserving fluid sheets"

- Cornelis *et al.*, 2014, EUROGRAPHICS,
  "IISPH-FLIP for incompressible fluids"

Basics of FLIP

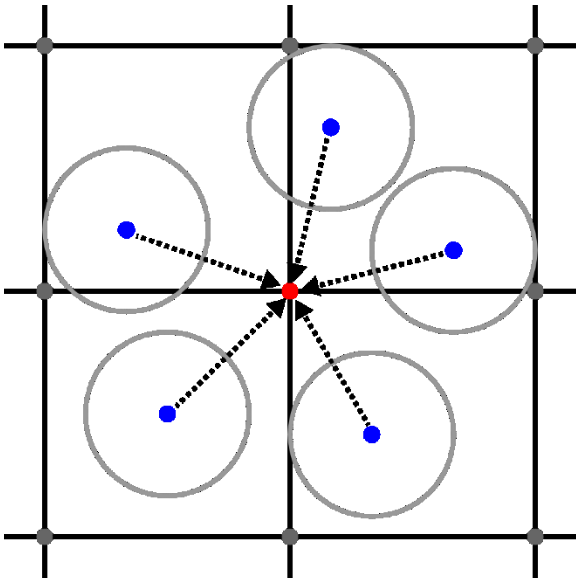
$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = \mathbf{g} - \frac{1}{\rho}\nabla p \qquad \nabla \cdot \mathbf{v} = 0$$

$\mathbf{v}$: velocity
$\mathbf{g}$: acceleration (gravity)
ρ: density
p: pressure

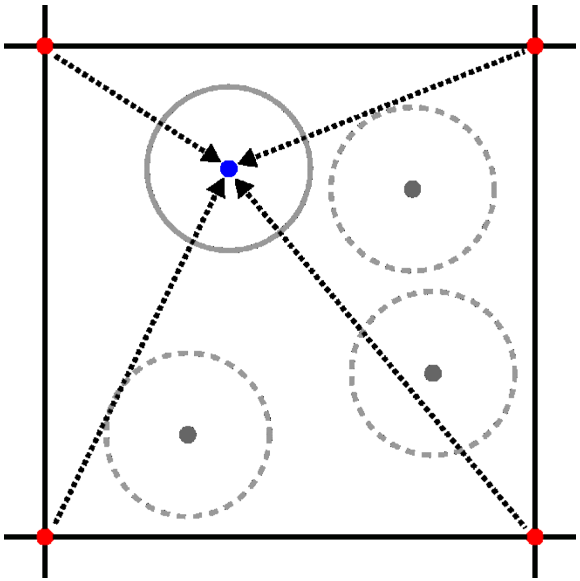
- Procedure
  1. Transfer the particle velocities to the grid points
  2. Solve the non-advection part on the grid
     - External forces
     - Incompressibility condition
  3. Update the particle velocity
  4. Move the particles
     - Grid's divergence-free velocity

Basics of FLIP (cont'd)

- Transfer scheme in FLIP



weighted average



bi- (or tri-) linear interpolation

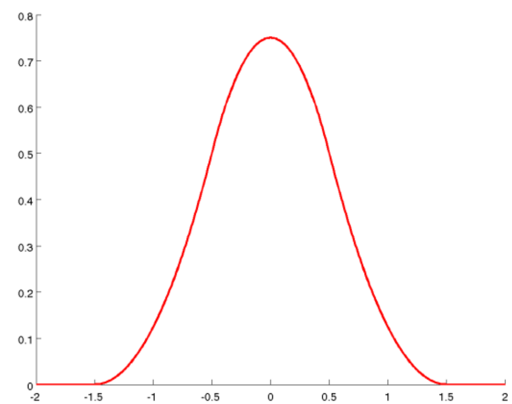
$$\mathbf{v}_\mathrm{g} = \frac{\sum_\mathrm{p} \omega_\mathrm{p} \mathbf{v}_\mathrm{p}}{\sum_\mathrm{p} \omega_\mathrm{p}}$$

$$\mathbf{v}_\mathrm{p} = \begin{cases} I(\mathbf{v}_\mathrm{g}, \mathbf{x}_\mathrm{p}), & \text{PIC} \\ \mathbf{v}_\mathrm{p} + I(\Delta\mathbf{v}_\mathrm{g}, \mathbf{x}_\mathrm{p}), & \text{FLIP} \end{cases}$$

Basis Function

- Basis function in the material point method

$$N(s) = \begin{cases} \frac{1}{2}s^2 + \frac{3}{2}s + \frac{9}{8}, & -\frac{3}{2} \le s < -\frac{1}{2} \\ -s^2 + \frac{3}{4}, & -\frac{1}{2} \le s < \frac{1}{2} \\ \frac{1}{2}s^2 - \frac{3}{2}s + \frac{9}{8}, & \frac{1}{2} \le s \le \frac{3}{2} \\ 0 & \text{otherwise} \end{cases}$$



$$\mathrm{N_g}(\mathbf{x}) = N\left(\frac{x}{h} - i\right) N\left(\frac{y}{h} - j\right) N\left(\frac{z}{h} - k\right)$$

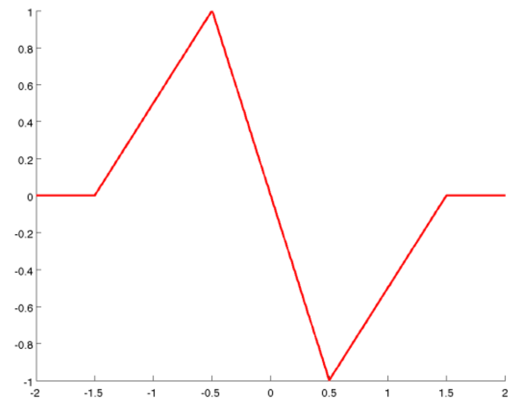
h:	(uniform) grid spacing
x, y, z:	particle position
i, j, k:	grid index

$$m_g = \sum_{\mathrm{p}} m_{\mathrm{p}} \mathrm{N_g}(\mathbf{x}_{\mathrm{p}})$$

Basis Function (cont'd)

- Derivative of the basis function

$$N_s(s) = \begin{cases} s + \dfrac{3}{2}, & -\dfrac{3}{2} \le s < -\dfrac{1}{2} \\ -2s, & -\dfrac{1}{2} \le s < \dfrac{1}{2} \\ s + \dfrac{3}{2}, & \dfrac{1}{2} \le s \le \dfrac{3}{2} \\ 0 & \text{otherwise} \end{cases}$$

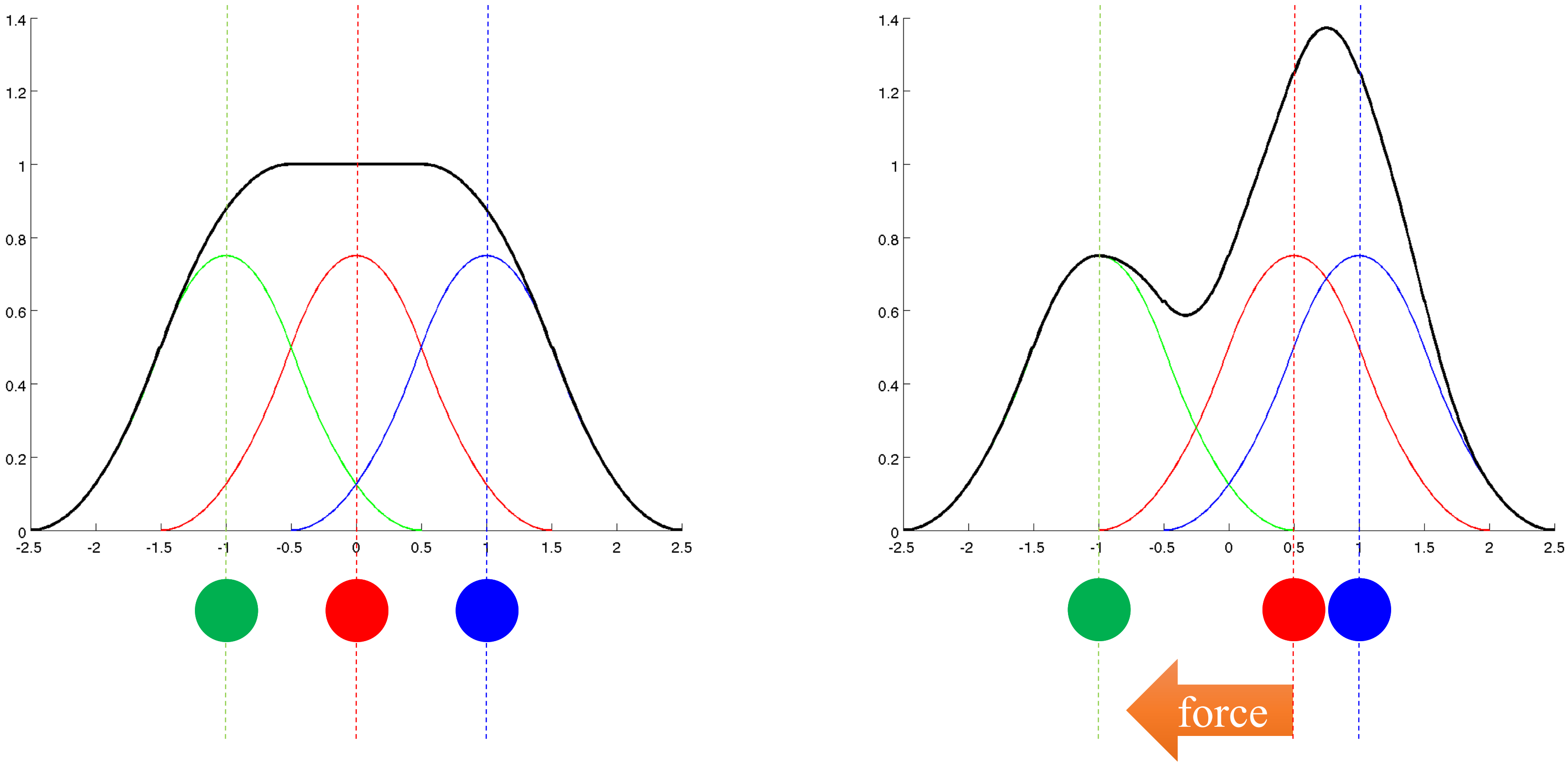


$$\nabla \mathrm{N_g}(\mathbf{x}) = \begin{bmatrix} N_x\left(\dfrac{x}{h} - i\right) N\left(\dfrac{y}{h} - j\right) N\left(\dfrac{z}{h} - k\right) \\ N\left(\dfrac{x}{h} - i\right) N_y\left(\dfrac{y}{h} - j\right) N\left(\dfrac{z}{h} - k\right) \\ N\left(\dfrac{x}{h} - i\right) N\left(\dfrac{y}{h} - j\right) N_z\left(\dfrac{z}{h} - k\right) \end{bmatrix}$$

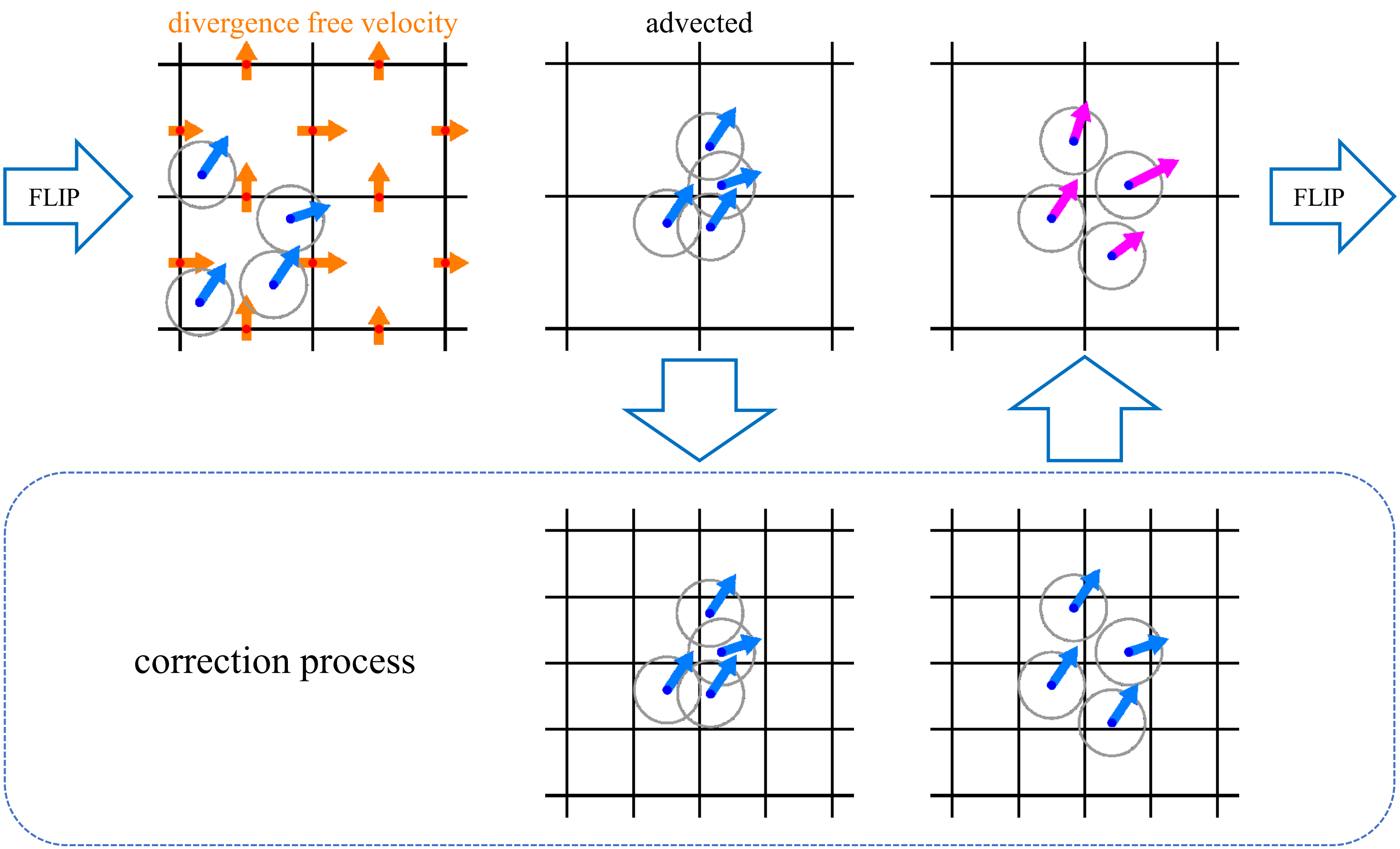
$$\mathbf{f}_\mathrm{g} = -\sum_\mathrm{p} p_\mathrm{p} \nabla \mathrm{N_g}(\mathbf{x}_\mathrm{p}) V_\mathrm{p}$$

Basis Function (cont'd)

- Transferred mass and correction force

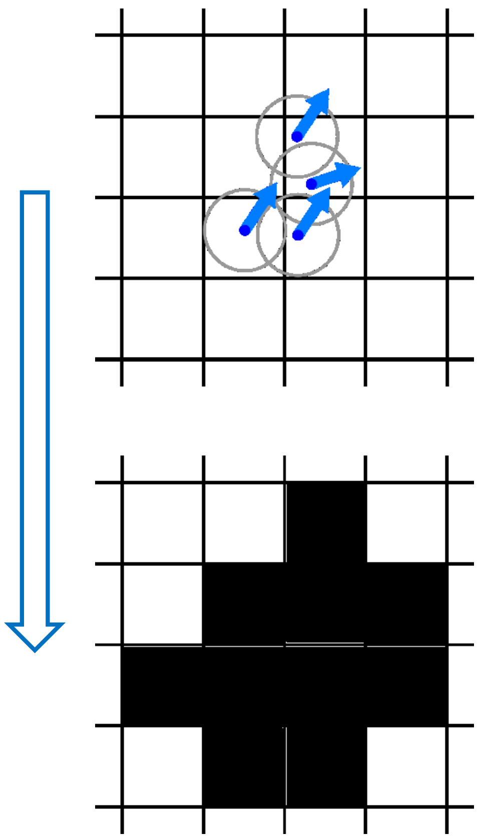


Simulation Overview



Particle Correction

- Grid-based particle correction



mass m_g

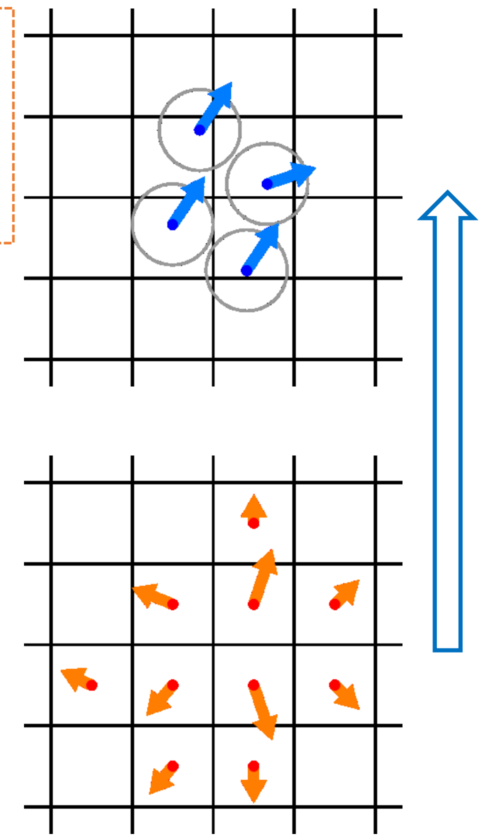
V_g, V_p: volume of a grid cell and a particle
ρ_0: rest density
k_p: stiffness coefficient
$N(\cdot)$: basis function

$$\rho_p = \frac{I(m_g, \mathbf{x}_p)}{V_g}$$

$$p_p = k_p\left(\frac{\rho_p}{\rho_0} - 1\right)$$

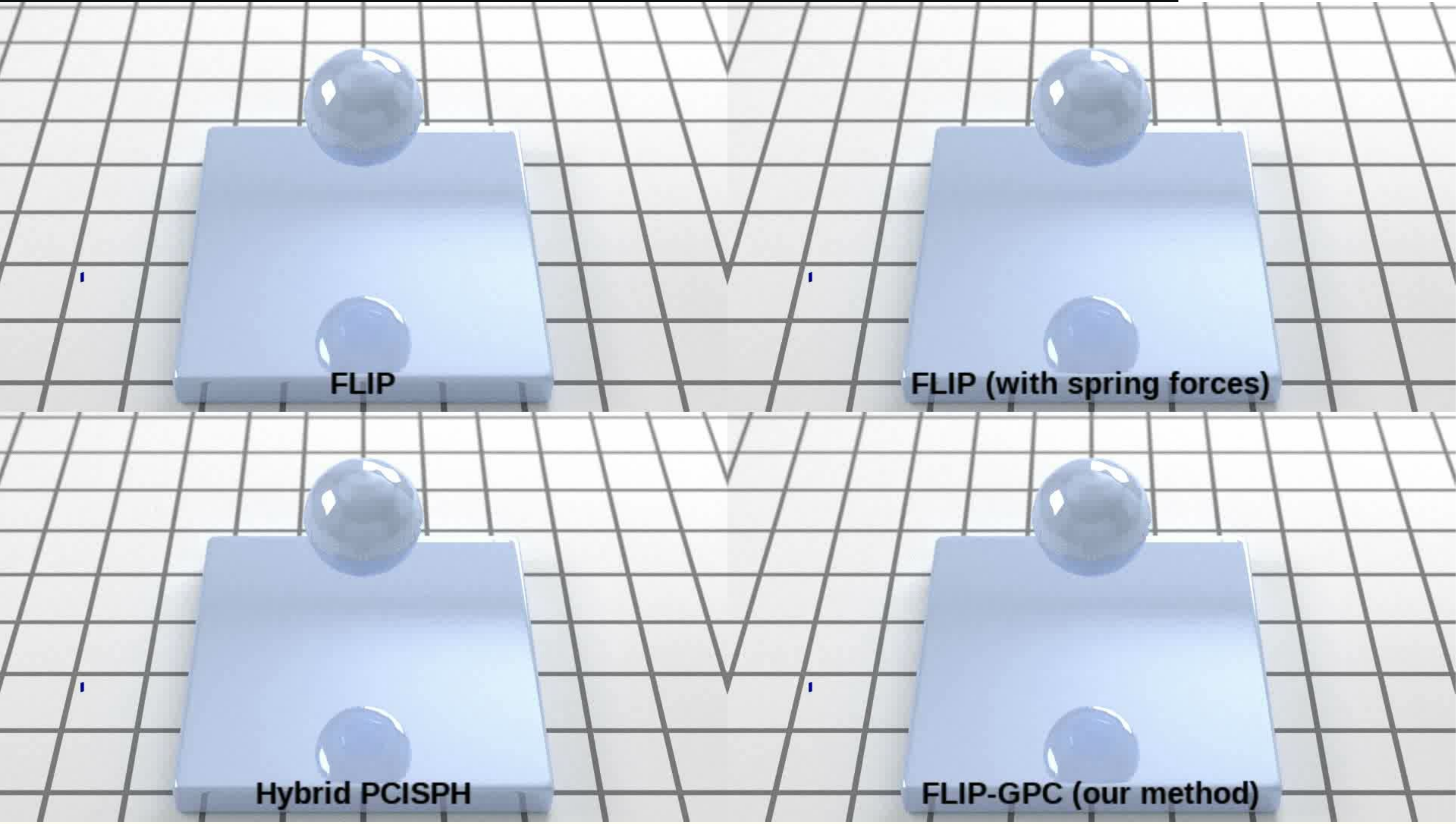
$$\mathbf{f}_g = -\sum_p p_p \nabla N(\mathbf{x}_p) V_p$$

pressure force



sub-grid velocity

Experimental Results



Three liquid balls: 64^3 grid, 2^3 sub-grid, and 200K~246K particles

Experimental Results (cont'd)

- Artificial tension effect

$$\rho_{\mathrm{p}} = \frac{I(m_{\mathrm{g}}, \mathbf{x}_{\mathrm{p}})}{V_{\mathrm{g}}} \qquad p_{\mathrm{p}} = k_p \left(\frac{\rho_{\mathrm{p}}}{\rho_0} - 1 \right)$$

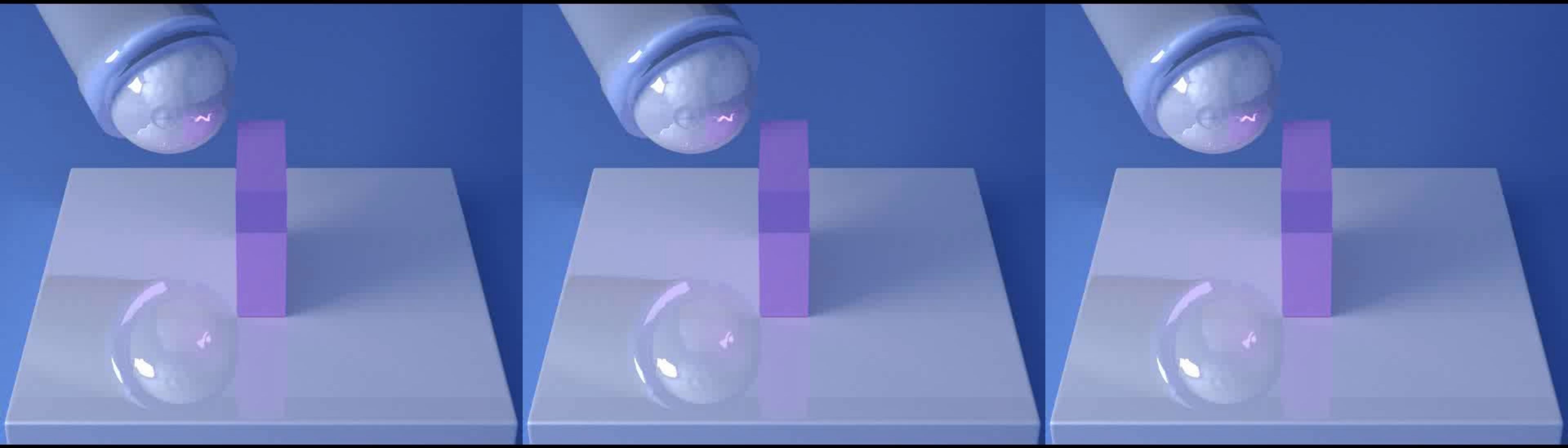
  - Free-surface region

- Solutions
  - Clamped pressure (non-negative pressure)
  - Ghost particles
    - [Schechter and Bridson, 2012, SIGGRAPH, "Ghost SPH for animating water"]
  - Near-density model
    - [Clavet *et al.*, 2005, SCA, "Particle-based viscoelastic fluid simulation"]

Experimental Results (cont'd)

Artificial tension: $128 \times 256 \times 128$ grid, 2^3 sub-grid, and 200K~246K particles

Experimental Results (cont'd)



FLIP (with spring forces)
21 sec/step

FLIP-GPC
15 sec/step

FLIP-GPC (clamped)
17 sec/step

Moving paddle: 128^3 grid, 2^3 sub-grid, and 2M~4M particles

Experimental Results (cont'd)

- Experimental environment
  - Intel Core i5-2500K 3.30 GHz processor
  - 32 GB memory
  - OpenMP for parallel computing
  - Cycle render engine in Blender for rendering

Conclusion

- Hybrid FLIP-GPC method
  - Using a grid-based correction process
  - Improving the particle distribution
  - Conserving volume (and mass)
  - Generating dynamic surface motions

- Future work
  - Applying adaptive approaches